LESSON 1

No Substitute for Hard Work

Evaluating Algebraic Expressions

Learning Goals

- Compare unknown quantities on a number line.
- Solve real-life and mathematical problems using algebraic expressions.
- Combine like terms to rewrite linear expressions and determine sums and differences.
- Write and evaluate algebraic expressions.
- Rewrite expressions in different forms in context to shed light on the relationship between quantities in a problem.

KEY TERMS

- variable
- algebraic expression
- linear expression
- constraint
- like terms
- numeric coefficient
- evaluate an algebraic expression

REVIEW (1–2 minutes)

Perform each operation.

1. $(-3)(6.6)$

2. $-3 + 6.6$

3. $-3 - 6.6$

4. $6.6 \div (-3)$

You have written and evaluated equivalent algebraic expressions with positive rational numbers.

How do you rewrite equivalent algebraic expressions and evaluate them over the set of rational numbers?

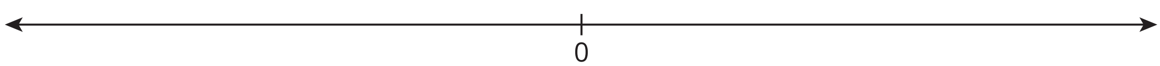
The Empty Number Line

Consider the list of six *variable* expressions.

x	$2x$	$3x$	$\frac{1}{2}x$	$-x$	$-\frac{1}{2}x$

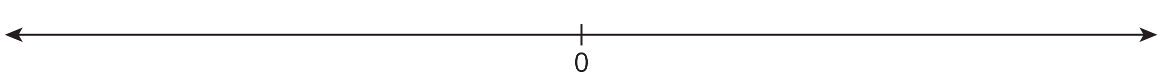
TAKE NOTE . . .
In algebra, a **variable** is a letter or symbol used to represent an unknown quantity.

1. With your partner, think about where you would place each expression and sketch your conjecture.



2. Compare your number line with another group's number line.
What is the same? What is different?

3. Your teacher will select students to place an index card representing each expression on the number line on the board. Record the locations agreed upon by the class.



Algebraic Expressions

HABITS OF MIND
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

In this lesson, you will explore the relationship between unknown quantities by writing and evaluating *algebraic expressions*. An **algebraic expression** is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Each expression in the Getting Started is an algebraic expression. They are also *linear expressions*. A **linear expression** is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

ASK YOURSELF . . .
How could you verify the placement of the expressions on the number line?

WORKED EXAMPLE

These are some additional examples of linear expressions:

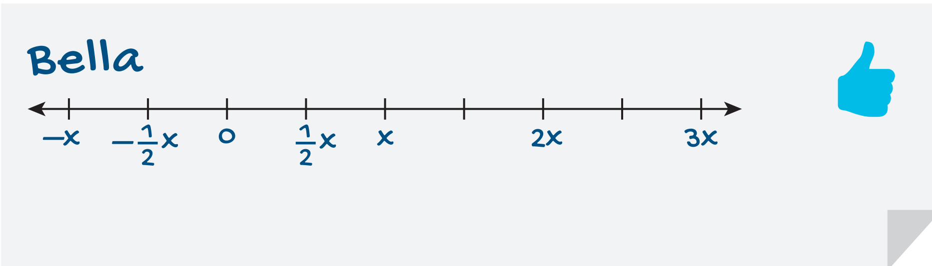
Examples of **linear** expressions	Examples of **nonlinear** expressions
$\frac{1}{2}x + 2$	$3x^2 + 5$
$\frac{x}{3} + 1$	$-\frac{1}{2}xy$
$-1 + 3x + \frac{5}{2}x - \frac{4}{3}$	$\frac{1}{x}$
$4y$	$x^2 + 2x + 1$

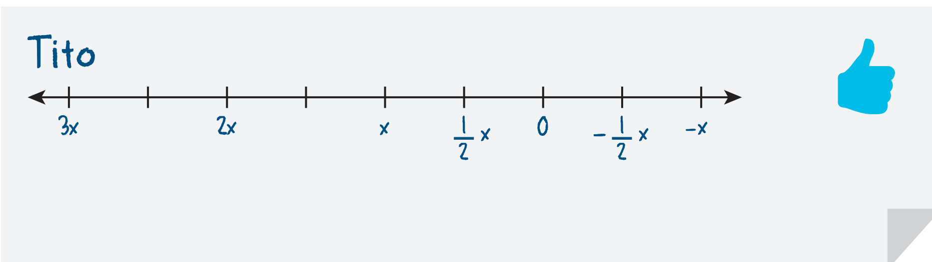
1. Provide a reason why each expression does not represent a linear expression.

Let's revisit how you may have plotted the expressions in the previous activity. The directions did not specify the possible values for x. When you plotted each expression, did you think about the set of all possible values of x or just the set of positive x-values?

In mathematics, it is sometimes necessary to set *constraints* on values. A **constraint** is a condition that a solution or problem must satisfy. It can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Analyze the number lines created by Bella and Tito using the expressions from the Getting Started.





2 Compare and contrast each representation.

(a) Identify the set of *x*-values that make each number line true. **Write each constraint as an inequality.**

(b) Select a value for *x* from your set of possible values and substitute that value for *x* into each expression to verify the plotted locations are correct.

(c) Compare your values from part (b) with your classmates. **Do you have the same values? If not, what does that mean?**

THINK ABOUT . . .

One strategy to verify your placement of the cards is to substitute values for the variable *x* into each expression.

Combining Like Terms

HABIT OF MIND
- Attend to precision.

Revisit the algebraic expressions from the Getting Started, along with two other algebraic expressions: $-2x$ and $-3x$.

x	$2x$	$3x$	$\frac{1}{2}x$	$-x$	$-\frac{1}{2}x$	$-2x$	$-3x$

These algebraic expressions are *like terms*. **Like terms** are parts of an algebraic expression that have the same variable raised to the same power. The *numeric coefficients* of the variable may be different.

A **numeric coefficient** is a number multiplied by a variable expression. For example, in the expression $-2x$, the number (-2) is a numeric coefficient. In the expression x, the numeric coefficient is 1. And in the expression $3(x + 1)$, the numeric coefficient is 3.

TAKE NOTE . . .
Constants in an algebraic expression are like terms because you can write them as the same variable expression with a power of 0.

WORKED EXAMPLE

Consider the expression $2x + 3x$.

You can rewrite this expression by combining like terms.

You combine like terms by adding the numeric coefficients.

$$2x + 3x = 5x$$

You can use the Distributive Property to justify this procedure.

$$2x + 3x$$
$$(2 + 3)x$$
$$5x$$

1. Combine like terms to rewrite each expression.

(a) $x + 2x$

(b) $x + \frac{1}{2}x$

(c) $x + \frac{-1}{2}x$

(d) $-3x + 2x$

(e) $-3x + -2x$

(f) $x + -x$

(g) $-\frac{1}{2}x + \frac{1}{2}x$

(h) $-3x + -2x + x$

To **evaluate an algebraic expression**, you substitute each variable in the expression with a number or numeric expression and then perform all possible mathematical operations.

WORKED EXAMPLE

You can evaluate expressions to verify their equivalence. Select any value for x, substitute that value into each expression, and evaluate.

Verify that $2x + 3x = 5x$.

Suppose $x = \mathbf{4}$.

$$2(\mathbf{4}) + 3(\mathbf{4}) \stackrel{?}{=} 5(\mathbf{4})$$
$$8 + 12 \stackrel{?}{=} 20$$
$$20 = 20$$

Suppose $x = \mathbf{-4}$.

$$2(\mathbf{-4}) + 3(\mathbf{-4}) \stackrel{?}{=} 5(\mathbf{-4})$$
$$-8 + -12 \stackrel{?}{=} -20$$
$$-20 = -20$$

2 Use $x = 4$ and $x = -4$ to evaluate each algebraic expression in Question 1 and verify your answers.

a $x + \frac{-1}{2}x = \frac{1}{2}x$

b $-3x + -2x = -5x$

c $x + -x = 0$

3 Evaluate each expression for the given values.

(a)

h	$-2h - 7$
2	
−1	
8	
−7	

TAKE NOTE . . .
Use parentheses to show multiplication, like $-2(-1) - 7$.

(b)

a	−12	−10	−4	0
$\frac{1}{4}a + 6$				

(c) Evaluate the expression $-\frac{1}{5}y + 3\frac{2}{5}$ using the set $\{-5, -1, 0, 15\}$. Write the results as a set of numbers.

Combining Like Terms with Decimal and Fractional Coefficients

HABIT OF MIND
- Attend to precision.

You can combine like terms to rewrite expressions more efficiently.

> Consider each situation to determine prices with discounts and with sales tax.

Suppose a new toy that regularly costs \$26.99 is on sale for $\frac{3}{4}$ off.

1. Write an expression to represent the price of the toy, p, minus $\frac{3}{4}$ of the price. Then, combine like terms to rewrite the expression.

TAKE NOTE . . .
Make sure you define your variables for each expression.

2. Explain what the rewritten expression means in terms of the original price of the toy.

A new shirt costs \$18.99. The sales tax is 5%.

3. Write an expression to represent the cost of the shirt, s, plus 5% of the cost. Then, combine like terms to rewrite the expression.

4. Explain what the rewritten expression means in terms of the original cost of the shirt.

5. Write an algebraic expression with the fewest terms to represent each situation.

   a. You give an 18% tip for a meal. What expression represents the total cost with tip?

   b. A pair of shoes sells for $\frac{1}{4}$ off. What expression represents the total cost after the discount?

   c. A store discounts a new bike by 35%. What expression represents the total cost?

Business Extras

Katie starts a limousine rental company. As part of her research, Katie discovers that she must charge a 7% sales tax to her customers in addition to her rental fees.

1. Write an algebraic expression that represents how much tax Katie should collect for any amount of rental fee.

Katie also discovers that most limousine rental companies collect a flat gratuity from customers in addition to the rental fee. Katie decides to collect a gratuity of $35 from her customers.

2. Write an expression that represents the total amount of additional money Kate collects for tax and gratuity.

3. Write an expression that represents the total cost of any rental.

4. Use one of your expressions to calculate the amount of tax and gratuity Katie should collect for a rental fee of $220.

5. Use one of your expressions to calculate the total cost of a rental for a rental fee of $365.

Use a separate piece of paper for your Journal entry.

JOURNAL

Explain the difference between a linear expression and an algebraic expression.

REMEMBER

You can combine like terms to rewrite algebraic expressions.

You can evaluate an algebraic expression by substituting a value for the variable and then performing all possible mathematical operations.

PRACTICE

Rewrite each expression by combining like terms, if possible.

1. $6x + 4x$
2. $-5y + 2y$
3. $9m - 7m + 13$
4. $4a + 8b$

Evaluate each algebraic expression for the given quantity.

5. $-6.2x + 1.4x$, $x = -9.3$
6. $3\frac{1}{2}x - 5\frac{1}{3}x$, $x = \frac{2}{5}$

Go to LiveHint.com for help on the PRACTICE questions.

Write an algebraic expression with the fewest terms to represent each situation.

7. Tim lives $\frac{2}{3}$ as far from school as Felipe. Felipe walks to school and then walks to Tim's house after school. What expression represents the total distance Felipe walked?

8. A store marks up the price of an item by 20%. What expression represents the cost a customer pays for the item?

9. The area of Circle A is $\frac{1}{4}$ the area of Circle B. What expression represents the difference between the areas of Circle A and Circle B?

STRETCH Optional

Evaluate each algebraic expression for the given values.

1. $-3(2.1x - 7.9)$ for $x = -18.1, -0.3, 14.4$
2. $-9.8t^2 + 20t + 8$ for $t = -2, 0, 3.5$