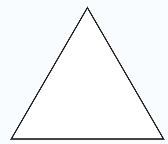
The Koch Snowflake

The Koch snowflake is a fractal created using equilateral triangles. It is based on the Koch curve, first mentioned in a 1904 paper by the Swedish mathematician Helge von Koch.

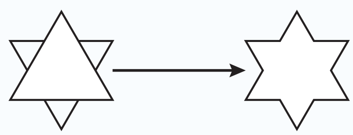
HABITS OF MIND

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

In **STAGE 0**, you begin with an equilateral triangle, such as the one shown. This is the first step in the creation of the Koch snowflake.



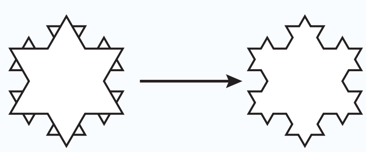
In **STAGE 1**, you divide each side of the triangle into thirds. Then, each middle segment becomes the base of a new equilateral triangle, as shown. Finally, you remove the middle segment.



1. How does a side length of a new triangle compare to a side length of the **STAGE 0** triangle?

2. When a side length of the **STAGE 0** figure is 1 unit, what is a side length of the **STAGE 1** figure?

In **STAGE 2**, you divide each side of the figure from **STAGE 1** into thirds, and the middle segments become the bases of new equilateral triangles. Then, you remove the middle segments.



You repeat this process on the remaining sides.

3. Describe the iterative process to create the Koch snowflake.

4 Let the side length at **STAGE 0** equal *s* units. Complete the table.

Stage (n)	Length of a Side	Number of Sides	Total Perimeter
0	s	3	$3s$
1			
2			
3			
4			
5			
n			

5 Identify the type of sequence represented by each characteristic.

(a) The length of a side

(b) The number of sides

(c) The total perimeter

6 Describe what happens to each characteristic as the iterative process continues.

(a) The length of a side

(b) The number of sides

(c) The total perimeter

7 Does this situation seem possible? **Explain your reasoning**.

8 Consider the equilateral triangle in **STAGE 0**. Each side length is 1 unit.

(a) Calculate the altitude. Leave your answer in radical form.

THINK ABOUT . . .
What two special right triangles will the altitude divide the equilateral triangle into?

(b) Calculate the area of the equilateral triangle. Leave your answer in radical form.

(c) What is the total area of the **STAGE 0** figure rounded to the nearest hundredth?

9 Consider one of the smaller triangles that you add to the **STAGE 0** figure.

(a) Calculate the altitude.

(b) Calculate the area of this triangle.

(c) Calculate the total area of the **STAGE 1** figure. Round your answer to the nearest hundredth.

TAKE NOTE . . .
To prevent rounding errors, leave your answers in radical form until the last step.

10 Consider one of the smaller triangles that you add to the **STAGE 1** figure.

(a) Calculate the altitude.

(b) Calculate the area of this triangle.

(c) Calculate the total area of the **STAGE 2** figure. Round your answer to the nearest hundredth.

11 Complete the table shown for **STAGE 0** through **STAGE 2**.

Stage	Number of New Triangles	Area of One New Triangle	Total Area in Radical Form	Total Area (nearest hundredth)
0				
1				
2				
3				

12 Use your table to answer each question.

(a) Predict the number of new triangles in the **STAGE 3** figure, the area of one new triangle, and the total area of the figure. **Explain your reasoning**. Record your results in the table.

(b) What happens to the number of new triangles as the stage number increases?

(c) What happens to the area of one new triangle as the stage number increases?

(d) What happens to the total area as the stage number increases? **Explain your reasoning**.

ACTIVITY 3

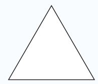
SUMMARY **The Koch snowflake is another example of a fractal.**

The Koch Snowflake

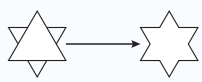
The Koch snowflake is a fractal created using equilateral triangles. It is based on the Koch curve, first mentioned in a 1904 paper by the Swedish mathematician Helge von Koch.

HABITS OF MIND
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

In **STAGE 0**, you begin with an equilateral triangle, such as the one shown. This is the first step in the creation of the Koch snowflake.



In **STAGE 1**, you divide each side of the triangle into thirds. Then, each middle segment becomes the base of a new equilateral triangle, as shown. Finally, you remove the middle segment.



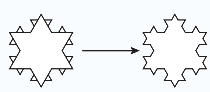
1. How does a side length of a new triangle compare to a side length of the **STAGE 0** triangle?

   The side length of a new triangle is one-third of the side length of the STAGE 0 triangle.

2. When a side length of the **STAGE 0** figure is 1 unit, what is a side length of the **STAGE 1** figure?

   A side length of the STAGE 1 figure is $\frac{1}{3}$ unit.

In **STAGE 2**, you divide each side of the figure from **STAGE 1** into thirds, and the middle segments become the bases of new equilateral triangles. Then, you remove the middle segments.



You repeat this process on the remaining sides.

3. Describe the iterative process to create the Koch snowflake.

   The iterative process to create the Koch snowflake is that for each stage, I divide every side of an equilateral triangle into thirds and remove the center segment and replace it with two segments of the same size, pointing outward.

Chunking the Activity

- Read and discuss the introduction
- Group students to complete 1 and 2
- Check in and share
- Group students to complete 3 and 4
- Check in and share
- Group students to complete 5–7
- Check in and share
- Group students to complete 8
- Check in and share
- Group students to complete 9 and 10
- Check in and share
- Group students to complete 11 and 12
- Share and summarize

LANGUAGE LINK

ELL TIP

Encourage students to use pictures to help describe the iterative process.

NOTES

ACTIVITY 3 Continued

4 Let the side length at **STAGE 0** equal *s* units. Complete the table.

Stage (n)	Length of a Side	Number of Sides	Total Perimeter
0	s	3	$3s$
1	$\left(\frac{1}{3}\right)^1 s$	12	$4s$
2	$\left(\frac{1}{3}\right)^2 s$	48	$\frac{48}{9}s$
3	$\left(\frac{1}{3}\right)^3 s$	192	$\frac{192}{27}s$
4	$\left(\frac{1}{3}\right)^4 s$	768	$\frac{768}{81}s$
5	$\left(\frac{1}{3}\right)^5 s$	3072	$\frac{3072}{243}s$
n	$\left(\frac{1}{3}\right)^n s$	$3 \cdot 4^n$	$3 \cdot \left(\frac{4}{3}\right)^n \cdot s$

5 Identify the type of sequence represented by each characteristic.

(a) The length of a side

The length of a side represents an infinite geometric sequence.

(b) The number of sides

The number of sides represents an infinite geometric sequence.

(c) The total perimeter

The total perimeter represents an infinite geometric sequence.

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Questions to Support Discourse

		TYPE
4	• How did you determine the number of sides in Stage 1? • Show how the pattern in the number of sides relates to snowflakes. • What patterns did you recognize in the table to construct the expressions for *n* sides?	Probing
5	• How do you know these are geometric sequences?	Seeing structure

ACTIVITY 3 Continued

6 Describe what happens to each characteristic as the iterative process continues.

(a) The length of a side

For each iteration, the length of a side is multiplied by $\frac{1}{3}$.
So, as n approaches infinity, the length of a side in the Koch snowflake approaches 0.

(b) The number of sides

For each iteration, the number of sides is multiplied by 4.
So, as n approaches infinity, the number of sides in the Koch snowflake approaches infinity.

(c) The total perimeter

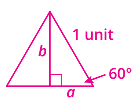
For each iteration, the total perimeter is multiplied by $\frac{4}{3}$.
So, as n approaches infinity, the total perimeter in the Koch snowflake approaches infinity.

7 Does this situation seem possible? **Explain your reasoning**.

It does not seem possible that the side lengths gets closer to zero, but the perimeter continues to grow.

8 Consider the equilateral triangle in **STAGE 0**. Each side length is 1 unit.

(a) Calculate the altitude. Leave your answer in radical form.



$2a = 1$

$a = \frac{1}{2}$

$b = \frac{\sqrt{3}}{2}$ unit

THINK ABOUT . . .
What two special right triangles will the altitude divide the equilateral triangle into?

(b) Calculate the area of the equilateral triangle. Leave your answer in radical form.

$A = \frac{1}{2}(1)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4}$ square unit

(c) What is the total area of the **STAGE 0** figure rounded to the nearest hundredth?

$A = \frac{\sqrt{3}}{4}$

$\approx$ 0.43 square unit

Questions to Support Discourse

		TYPE
6	• How can you tell whether the values approach zero or infinity?	Seeing structure
8	• How did you determine the altitude of the triangle?	Probing

NOTES

TOPIC 4

NOTES

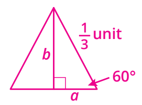
Student Look-Fors

Utilizing relationship skills by communicating clearly and listening well

ACTIVITY 3 Continued

9 Consider one of the smaller triangles that you add to the **STAGE 0** figure.

ⓐ Calculate the altitude.



$2a = \frac{1}{3}$

$2a = \frac{1}{6}$

$b = \frac{\sqrt{3}}{6}$ unit

ⓑ Calculate the area of this triangle.

$A = \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{6}\right)$

$= \frac{\sqrt{3}}{36}$ square unit

ⓒ Calculate the total area of the **STAGE 1** figure. Round your answer to the nearest hundredth.

$A = \frac{\sqrt{3}}{4} + 3\left(\frac{\sqrt{3}}{36}\right)$

$= \frac{9\sqrt{3}}{36} + \frac{3\sqrt{3}}{36}$

$= \frac{12\sqrt{3}}{36} = \frac{\sqrt{3}}{3}$

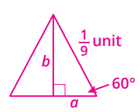
$\approx$ 0.58 square unit

TAKE NOTE . . .

To prevent rounding errors, leave your answers in radical form until the last step.

10 Consider one of the smaller triangles that you add to the **STAGE 1** figure.

ⓐ Calculate the altitude.



$2a = \frac{1}{9}$

$a = \frac{1}{18}$

$b = \frac{\sqrt{3}}{18}$ unit

ⓑ Calculate the area of this triangle.

$A = \frac{1}{2}\left(\frac{1}{9}\right)\left(\frac{\sqrt{3}}{18}\right)$

$= \frac{3}{324}$ square unit

ⓒ Calculate the total area of the **STAGE 2** figure. Round your answer to the nearest hundredth.

$A = \frac{\sqrt{3}}{3} + 12\left(\frac{\sqrt{3}}{324}\right)$

$= \frac{108\sqrt{3}}{324} + \frac{12\sqrt{3}}{324}$

$= \frac{120\sqrt{3}}{324} = \frac{10\sqrt{3}}{27}$

$\approx$ 0.64 square unit

Questions to Support Discourse

		TYPE
9	• How many times smaller is the triangle in Stage 1 compared to the triangle in Stage 0? Why? • How did you calculate the total area?	Probing
10	• How do your calculations compare to those for the previous stage?	Probing

ACTIVITY 3 Continued

11 Complete the table shown for **STAGE 0** through **STAGE 2**.

Stage	Number of New Triangles	Area of One New Triangle	Total Area in Radical Form	Total Area (nearest hundredth)
0	1	$\frac{\sqrt{3}}{4}$	$\frac{\sqrt{3}}{4}$	0.43
1	3	$\frac{\sqrt{3}}{36}$	$\frac{\sqrt{3}}{3}$	0.58
2	12	$\frac{\sqrt{3}}{324}$	$\frac{10\sqrt{3}}{27}$	0.64
3	48	$\frac{\sqrt{3}}{2916}$	$\frac{94\sqrt{3}}{243}$	0.67

12 Use your table to answer each question.

(a) Predict the number of new triangles in the **STAGE 3** figure, the area of one new triangle, and the total area of the figure. **Explain your reasoning**. Record your results in the table.

The number of new triangles in the STAGE 3 figure is four times the number of new triangles in the STAGE 2 figure. The area of a new triangle is $\frac{1}{9}$ of the area of a new triangle in the STAGE 2 figure.

Add the area of each new triangle to the area of the STAGE 2 figure to calculate the total area.

$$A = \frac{10\sqrt{3}}{27} + 48\left(\frac{\sqrt{3}}{2916}\right)$$
$$= \frac{1080\sqrt{3}}{2916} + \frac{48\sqrt{3}}{2916}$$
$$= \frac{1128\sqrt{3}}{2916} = \frac{94\sqrt{3}}{243}$$
$$\approx 0.67 \text{ square unit}$$

(b) What happens to the number of new triangles as the stage number increases?

The number of new triangles quadruples as the stage number increases.

(c) What happens to the area of one new triangle as the stage number increases?

The area of one new triangle decreases by a factor of $\frac{1}{9}$ as the stage number increases.

(d) What happens to the total area as the stage number increases? **Explain your reasoning**.

The total area increases as the stage number increases. The total area grows slowly as the area of each new triangle gets smaller because the number of new triangles increases.

Lesson 3 > This Is the Title of This Lesson 741

Questions to Support Discourse

		TYPE
12	• How did you use patterns to complete the table for Stage 3? • Why doesn't the total area column have a recognizable pattern?	Probing

NOTES

TOPIC 4